obtained from the initial function u_0 by an area-preserving diffeomorphism of M onto itself (by an "*incompressible fluid motion*").

The extremal function u is smooth if the smooth initial mountain $u_0 : B^2 \to \mathbb{R}$, vanishing on the boundary of the ball, has just one nodegenerate (Morse) maximum inside the ball. In this case, the extremal function u is the symmetrization of u_0 (depending only on the distance from the center of the ball).

But for the initial smooth mountain u_0 having (like the Elbrus mountain) two local maxima separated by a saddle point, the extremal function seems to have a singularity of type $|x|$ along a curve with unknown extremal function singularities at its endpoints. The problem is to study such singularities for generic u_0.

2002-8. The (C,B,A)-*permutation* of the set $\{1,2,\ldots,n\}$ transports to the last place the subset $A = \{1,2,\ldots,a\}$ preceded by the transported set $B = \{a+1, \ldots, a+b\}$ while the starting position is occupied by $C = \{a+b+1, \ldots, n\}$.

Some of these $(n-1)(n-2)/2$ permutations permute *cyclically* (like the addition of a constant to the residues mod n), and some of these cyclic permutations are *transitive* (like the addition of the constant 1).

Find the proportion of both the cyclic and the transitive cyclic permutations among the (C,B,A)-permutations for large n.

More generally, starting from a permutation of k elements, one defines a permutation of the set $\{1,\ldots,n\}$ from its decomposition into k segments $\{a_i + 1, \ldots, a_{i+1} - 1\}$. The problem is to study the statistics of the Young diagrams formed by the cycle lengths of the resulting permutations, for the case of large n and random decompositions of n into k parts.

2002-9. A mapping $\mathbb{C}^n \to \mathbb{C}^n$ (or $\mathbb{C}\mathrm{P}^n \to \mathbb{C}\mathrm{P}^n$) is called a *pseudocomplex* mapping if it sends complex subspaces to complex subspaces (one may consider separately the cases of vector, affine or projective subspaces—all the three versions are interesting).

A real diffeomorphism $\mathbb{C}\mathrm{P}^2 \to \mathbb{C}\mathrm{P}^2$ is pseudocomplex if and only if either it, or its product with the complex conjugation, is a complex projective mapping (and similarly for the other versions and other n's).

Do there exist other pseudocomplex homeomorphisms? Other pseudocomplex bijections?

These questions should have been studied by Hilbert as a part of axiomatic projective geometry, but his school seems to have missed these foundational problems.

▽ 2002-8

This is Problem 8 in paper [1] which is also included in book [2].

[1] ARNOLD V. I. Problems to the Seminar: 15 January 2002. CEREMADE (UMR 7534), Université Paris-Dauphine, № 0216, 16/05/2002.

[2] ARNOLD V. I. What Is Mathematics? Moscow: Moscow Center for Continuous Mathematical Education Press, 2002, 104 pp.

△ 2002-8 – *V. I. Arnold*

Among the 45 (C,B,A)-permutations for $n = 11$, there are 22 cyclic permutations (which are all transitive), and for the preceding values of n the numbers of cyclic and of transitive cyclic permutations form two strange nonmonotone sequences.

Among all the $n!$ permutations of n elements, those which are transitively cyclic form a small part (their number is $(n-1)!$), and the number of all cyclic ones seems to be asymptotically equivalent to the same $1/n$th part of the total number of permutations as that of transitively cyclic ones.

Thus, it seems that *the statistic of the (C,B,A)-permutations is essentially different from that of the generic ones* (and one might study the same difference for other permutations of several letters acting on a large set $\{1,\dots,n\}$).

All these questions had been formulated in 1958 (see problem *1958-1*) at my Moscow seminar as simplified models for the segments interchange mapping problem in the theory of dynamical systems, which was later published in my 1963 long paper [1] on Hamiltonian dynamics. The present day state of this problem (studied especially by M. L. Kontsevich and A. V. Zorich) is described in Zorich's recent paper [2].

Kontsevich has recently attempted to claim that the interchange of three segments is always rotation equivalent. Problem *2002-8* requires the study of the *frequency* of the counterexamples to this "equivalence" (the existence of such counterexamples being, of course, known to me in the 1950s while I was inventing the segments interchange problem).

[1] ARNOLD V. I. Small denominators and problems of stability of motion in classical and celestial mechanics. *Russian Math. Surveys*, 1963, **18**(6), 85–191.

[2] ZORICH A. How do the leaves of a closed 1-form wind around a surface? In: Pseudoperiodic Topology. Editors: V. Arnold, M. Kontsevich and A. Zorich. Providence, RI: Amer. Math. Soc., 1999, 135–178. (AMS Transl., Ser. 2, 197; Adv. Math. Sci., 46.)